Mixing Rates in Stirred Tanks

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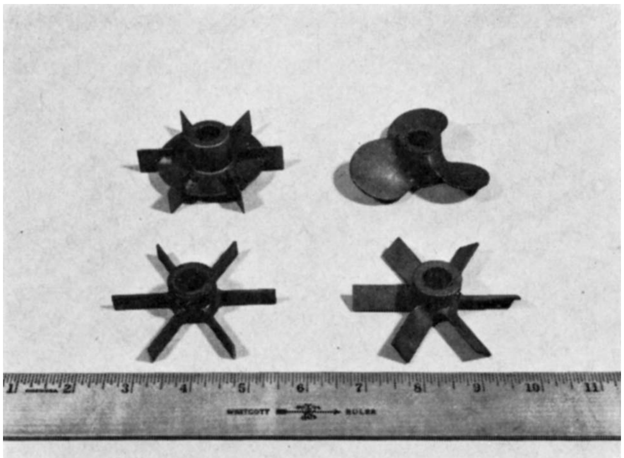

Fig. 1. Mixing impellers.

There is much interest in the rate of mixing in agitated tanks because of the wide application of this type of equipment in chemical processing. A number of studies, both theoretical and experimental, have been published which attempted to examine the factors which influence the mixing rates in impeller-agitated tanks (*1* to *17*). Hyman (*18*) wrote an excellent review of the work on mixing rates published prior to 1959. Beek and Miller (*22*) described three simultaneous stages of the turbulent mixing of miscible fluids.

The first stage is the distribution of one fluid through the other to make the time average concentrations uniform throughout the mixed fluids. In many blending operations this is the only important stage of the mixing process. In the second stage, portions of the fluid are broken into smaller portions which are intermingled, and thus generate additional contact area between regions of differing concentrations. The final stage of mixing occurs by molecular diffusion on a scale smaller than that which may be influenced by the turbulent motion. All three stages of mixing may be important in chemically reacting systems.

The present investigation deals primarily with the rate of mixing in the first stage of the mixing process described above.

EXPERIMENTAL

Apparatus

The apparatus used consisted of a 9⅝-in. diam. cylindrical glass tank fitted with feed and discharge ports for continuous flow of liquid through the tank, a variable speed agitator with a variety of impellers, and a constant-head feed reservoir to provide a steady flow of liquid to the mixing tank. The rate of the liquid feed to the mixing tank was measured with a calibrated rotameter.

Figure 1 illustrates the mixing impellers. The disk-and-vane turbine dimensions conformed to those used by other investigators (*19*); the propeller was a standard square-pitch design (*20*), and the straight-blade and pitched-blade turbines had blade width-to-turbine diameter ratios of 1 to 8.

The rate of mixing was determined by recording the variation of concentration at a point near the discharge from the tank after a small volume of tracer was introduced into the mixing tank in the feed stream. An electrical conductivity probe measured the electrolyte tracer concentration.

One electrode of the conductivity probe consisted of a helical coil, ¼-in. long and ¼-in. in diam., of platinum wire, and the other electrode consisted of a straight wire mounted at the axis of the helix. This construction permitted the free flow of liquid through the cell, but limited the conductivity measurement to the small volume of liquid within the helix.

A bridge measured the resistance of the conductivity probe, and a recorder connected across the bridge through a suitable amplifier recorded the resistance variation of the conductivity probe with time. Since the resistance of the conductivity cell was a linear function of the tracer concentration in the range of concentrations, the recording was indicative of the concentration-time history at the point in the mixing tank where the conductivity cell was located.

Procedure

The mixing tank was fitted with two feed lines with valves arranged so that the liquid flowing to the stirred tank could be switched from one line to the other with only a momentary interruption. One of the feed lines contained a small U-tube, located near the tank, into which a small volume of tracer could be placed when no liquid was flowing through that line.

After the system was adjusted to steady state operation with city water flowing into the stirred tank through the second feed line, the valves in the feed lines were suddenly switched, and the water flow was diverted through the first feed line and the tracer was swept into the tank. Electrical contacts on the feed valves were broken when the valves were switched, and a mark was made on the recorder tracing indicating the instant of tracer introduction.

Interpretation of Recording

Figure 2 is a sketch of a typical chart recording obtained during the test described above. The three *mixing times* illustrated are defined as follows:

t_1 is the time required for the trace to be displaced from the null position by 10% of the well-mixed concentration C_a. Thus t_1 is proportional to the time required for the first particle of tracer to move from the feed point to a point near the outlet of the tank.

t_2 is the time corresponding to the maximum concentration measured.

t_3 is the time, called the *terminal mixing time*, required for the trace to approach within ± 5% of C_a.

It was established experimentally that vertical distances on the recorder chart were directly proportional to concentration; therefore, the 5% deviation required to meas-

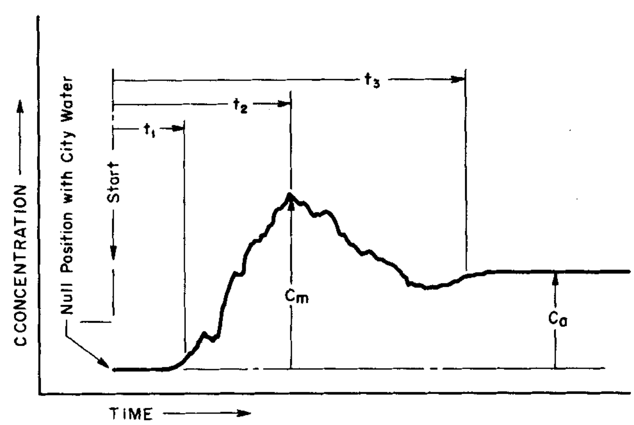

Fig. 2. Typical chart recording.

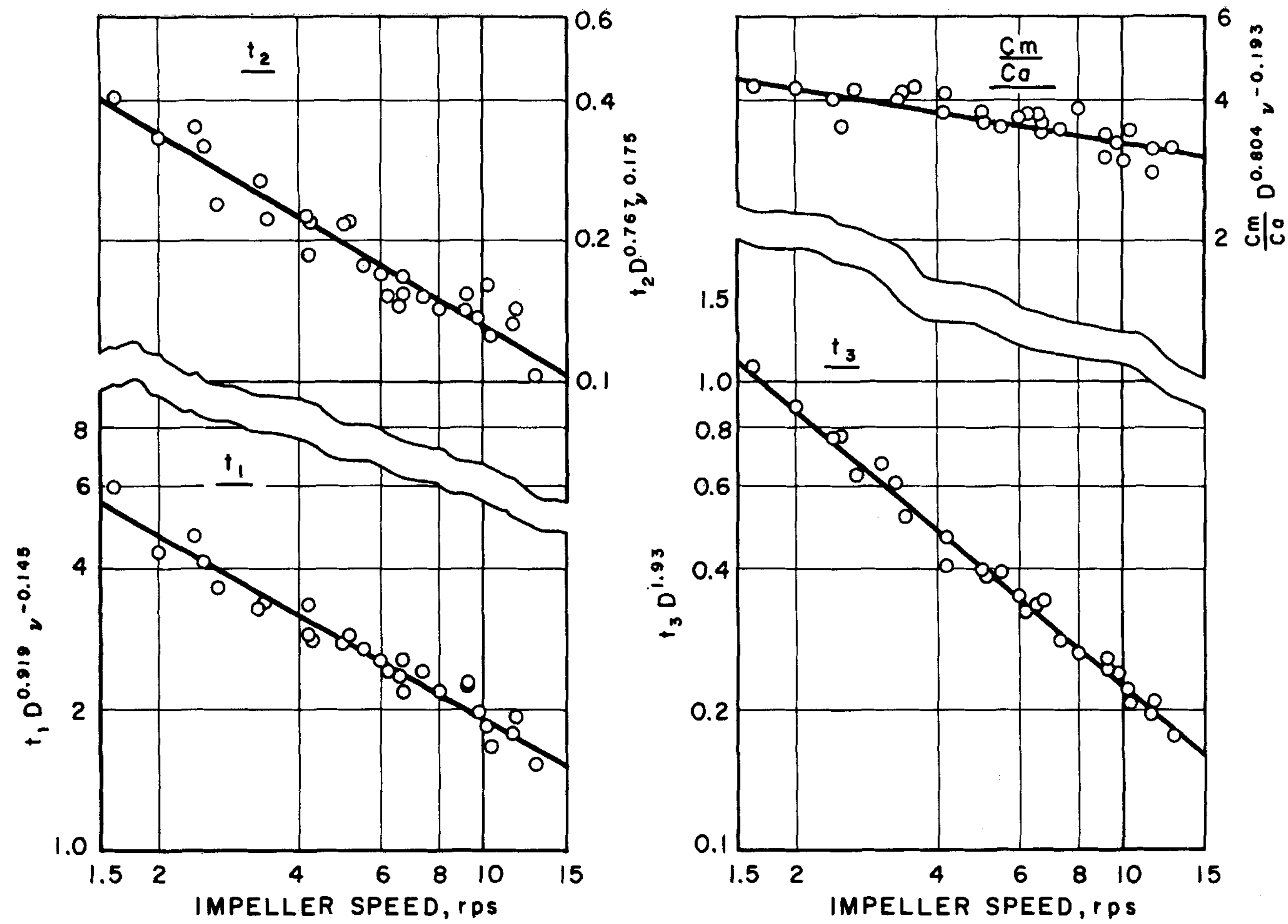

Fig. 3. Mixing time correlation, disk and vane turbine.

ure t_3 was obtained by direct measurement on the chart. The ratio C_m/C_a, which equals the ratio of the maximum concentration to the concentration after the tracer is well mixed, is indicative of the degree of dispersion attained as the slug of tracer passes through the mixing tank.

Although the experimental data were collected with a flow system, the observed mixing times are equivalent to mixing times measured in batch experiments. This becomes clear when one considers that the average residence time of the water flowing through the mixing tank was of the order of one hundred times greater than the terminal mixing times observed. Therefore, the tracer was uniformly dispersed in the tank before an appreciable fraction of the tracer was discharged from the mixing tank in the effluent stream.

The mixing times described above were obtained as averages of the results of five or more test runs at each set of experimental conditions. These data were independent of the concentration and composition of the electrolyte when the solution was sufficiently dilute to assure that the resistance of the conductivity cell was linear with concentration.

Experimental Conditions

All experimental work was carried out with the liquid depth equal to the tank diameter (9⅝-in.,) and the impeller centered in the tank with a clearance of 1 impeller diam. off the bottom. Four baffles, 1 in. wide, were mounted adjacent to the wall and extended to within ½ in. of the bottom of the tank. The variables studied and the range of values covered are listed below:

Impeller speed, n	100 to 800 rev./min.
Impeller diameter, D	2½ to 4 in.
Kinematic viscosity, ν	9.7×10^{-6} to 20×10^{-6} sq. ft./sec.
Reynolds number, nD^2/ν	10,000 to 90,000
Impeller type	See Figure 1
Feed rate	0.37 to 1.46 liters/min., corresponding to average residence times in the stirred tank of 8 to 31 min. Most data were collected at a feed rate of 0.73 liters/min. or an average residence time of 16 min.
Feed location	Feed inlet locations near the top of the liquid and near the impeller were tested. Discharge connections were always located on the side of the tank opposite the feed inlet.

The mixing times were corrected by the subtraction of an estimated time required for the tracer to flow into the mixing tank after the feed valve was opened. The time corrections used are tabulated as follows:

Feed rate liters/min.	Time correction sec.
0.37	0.85
0.73	0.43
1.46	0.21

The experimental data, including both the original and corrected mixing times, are tabulated in Table 1.*

* Table 1 has been deposited as document 7653 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25 D. C., and may be obtained for $1.25 for photoprints or for 35-mm. microfilm.

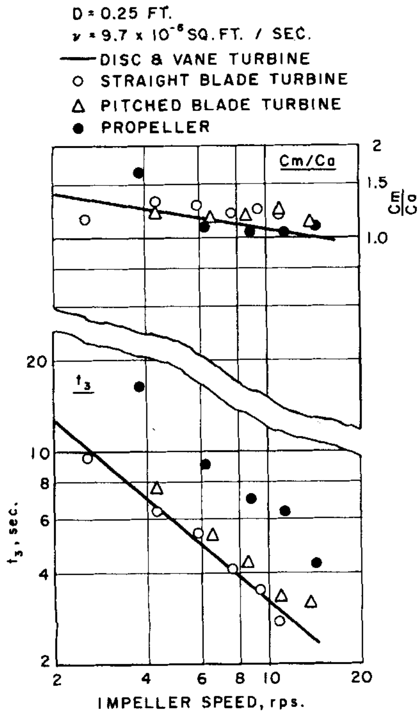

Fig. 4. Effect of impeller type on terminal mixing time and concentration ratio.

CORRELATION OF DATA

Disk-and-Vane Turbine

Twenty-eight runs were made with disk-and-vane turbines with a constant feed rate of 0.75 liters/min. and a single feed inlet location near the top of the mixing tank. (See data groups *C, H, I,* and *M* in Table 1.) The impeller speed, the impeller diameter, and the liquid viscosity were varied in these runs, and the three mixing times and the concentration ratio were correlated empirically by the equation

$$t_1, t_2, t_3 \text{ or } C_m/C_a = b_o n^{b_1} D^{b_2} \nu^{b_3} \tag{1}$$

The correlations are illustrated in Figure 3, and the values of b_o, b_1, b_2, and b_3 for each of the four correlations are tabulated in Table 2 with the 95% confidence limits for the exponents. The terminal mixing time, t_3, is independent of the liquid viscosity for the range of variables studied. Since Equation (1) is dimensional, the values of b_o in Table 2 apply only when the dimensions of the variables n, D, and ν are expressed in feet and seconds.

The form of the correlation was selected to show clearly the effects of the independent variables on the mixing times and the concentration ratio. A comparison of these data with data reported by other investigators and the application of the data are discussed below.

Effect of Impeller Type

The effect of the impeller type on the terminal mixing time, t_3, and the concentration ratio, C_m/C_a, is illustrated in Figure 4. (See data groups *J, K,* and *L* in Table 1.) There is little difference in terminal mixing times observed for the three types of turbines; however, the terminal mixing times for the propeller were approximately 100% greater than for the turbines. Results for mixing times t_1 and t_2 were similar to those shown in Figure 4. These results are significant because there are relatively large differences in the power required by turbines to produce approximately the same terminal mixing times.

The data for the concentration ratio show that the impeller type has little effect on the dispersion of the tracer as measured by C_m/C_a.

Effect of Feed Rate

The feed rate was varied from 0.37 liters/min. to 1.46 liters/min. to determine the effects of the feed rate on the observed mixing times (data groups *C, D,* and *E* in Table 1). These data failed to show any general effect of feed rate; however, this is not surprising since the terminal mixing times in all cases were a fraction of the average residence time of the feed in the tank. The work of Vande Vusse (*11*) and MacDonald and Piret (9) indicate that mixing times are not related to feed rate in a simple way.

Effect of Feed Location

Most of the data were collected with the feed inlet located near the top of the mixing tank and the discharge located near the impeller. A few runs were made with the feed and discharge connections reversed (data groups *F* and *G* in Table 1) and with the 2.5-in. diam. disk-and-vane turbine.

The mixing time data for the feed connection near the impeller were less reproducible than the remainder of the data, and the results are not highly significant. Qualitatively, the feed location made little difference for the lower impeller speeds; however, for the higher impeller speeds, the feed inlet location near the impeller gave faster initial dispersion of the tracer (that is, smaller values of t_1 and t_2) with little effect on the terminal mixing time.

COMPARISON WITH DATA OF OTHER INVESTIGATORS

The mixing time data reported by a number of investigators are summarized in Figure 5 for comparison with the author's data. The dimensionless group plotted as the ordinate of Figure 5 is the group used by Norwood and Metzner (*14*) to correlate their data for baffled turbines. The correlations of the other investigators have been converted to the form shown in Figure 5 for convenience in comparisons.

Norwood and Metzner's comprehensive study of flow patterns and batch mixing rates for baffled, turbine-agitated tanks covered a fivefold range of mixing Reynolds numbers up to a value of about 10^5. The exponents on the variables n and D reported by Norwood and Metzner compare well with the exponents listed in Table 2 for the terminal mixing times. Both studies show that the terminal mixing times are essentially independent of the liquid viscosity for Reynolds numbers greater than 10^4.

Kramers et al. (*13*) reported some experimental results for batch terminal mixing times in baffled and unbaffled tanks agitated with propellers and turbines. Their work was not extensive; however, the approximate range of mixing times they observed is indicated by the shaded region in Figure 5. These data apply to impeller locations in the mixing tank which gave minimum mixing times.

Fox and Gex (*10*) reported terminal mixing time data for the batchwise blending of miscible liquids in unbaffled tanks with eccentrically placed propellers and liquid jets. They found that minimum mixing times were

TABLE 2. REGRESSION COEFFICIENTS FOR EQUATION (1)

	t_1	t_2	t_3	C_m/C_a
b_0	6.93	0.506	1.554	4.74
b_1	-0.565 ± 0.093	-0.587 ± 0.137	-0.843 ± 0.061	-0.166 ± 0.077
b_2	-0.919 ± 0.312	-0.767 ± 0.421	-1.930 ± 0.194	-0.804 ± 0.260
b_3	0.145 ± 0.111	-0.175 ± 0.150	—	0.193 ± 0.093

observed for any set of operating conditions when the propeller or jet was placed to eliminate vortexing of the liquid. Their data for unbaffled propellers is plotted in Figure 5 for some typical tank-to-impeller diameter ratios.

VandeVusse (*11*) reported an extensive experimental investigation of the power requirements and terminal mixing times for the blending of miscible, nonviscous liquids in tanks agitated with paddles, propellers, or turbines. VandeVusse's experimental technique was sufficiently different from that reported by others that a direct comparison of mixing times in Figure 5 is not justified.

The data in Figure 5 provide a means for estimating the rate of distribution of a given element of fluid throughout the volume of a stirred tank for a number of commonly used impeller types. The studies of Fox and Gex and Norwood and Metzner are the most comprehensive (their data extend beyond the range shown in Figure 5) for unbaffled propellers and baffled disk-and-vane turbines. The work of Kramers et al. and VandeVusse provides qualitative confirmation of the data for turbines and propellers as well as data for other types of impellers. The data reported here confirm the results of Norwood and Metzner for disk-and-vane turbines, show that other types of turbines give terminal mixing times equivalent to disk-and-vane turbines in the turbulent regime, and indicate that mixing times for baffled propellers are intermediate between turbines and unbaffled propellers.

APPLICATIONS

Blending of Miscible Liquids

The terminal mixing time data shown in Figure 5 measure adequately the performance of a stirred tank in which only bulk blending is important. For continuous-flow, stirred-tank blenders, the terminal mixing time relative to the average residence time of the feed in the mixing tank determines the approach to uniformity of the effluent stream. MacDonald and Piret (*9*) indicate that the terminal mixing time should be less than 5% of the average residence time of the feed to assure that random deviations from the average product composition be held below 5%.

The reader is referred to published sources for data on the power requirements for a variety of mixing impellers (*18, 19*) and to the excellent discussions of the problem of the scale-up of a stirred tank (*14, 18*).

The power required to achieve a given mixing time varies considerably with the type of impeller. For example, for the three types of turbines tested in the work reported here, the relative power requirements for equal mixing times in the turbulent regime (Reynolds numbers $> 10^4$) are:

Turbine type	Relative power required
Disk-and-vane	5.2
Straight blade	1.9
Pitched blade	1.0

Mixing in Stirred Tank Reactors

Uniformity of composition on a microscopic scale is important for chemically reacting systems. Hughes (*21*) pointed out that turbulent motion is effective only to some minimum scale or eddy size. Below this minimum eddy size viscous shear forces prevent turbulent motion, and mixing on this scale occurs by molecular diffusion. An estimate based on the theory of isotropic turbulence may be made of the scale of the smallest eddies. Thus

$$l_s = \nu^{3/4}/\epsilon^{1/4} \qquad (2)$$

According to Hughes this estimate should apply reasonably well even for nonisotropic turbulence.

Manning and Wilhelm (*15*) reported their experimental measurements of concentration fluctuations in a turbine agitated tank in which a salt solution was mixed with water. It was estimated (*16*) that for Manning's apparatus the minimum scale of turbulent mixing in the region near the impeller was of the order of 10^{-3} to 10^{-4} cm. The scale of mixing calculated for the conditions of Manning's experiments with Equation (2) was of the order of 10^{-3} cm. The effective volume in which the energy dissipated was assumed to be the volume swept out by the turbine impeller.

Thus Equation (2) may be used to estimate the scale of the smallest eddies which are generated at the impeller. Hughes points out that the time required for the third stage of the mixing process, that is, the smoothing of the concentration differences within the smallest eddies by molecular diffusion, is of the order

$$t_m = l_s^2/2D_v \qquad (3)$$

Thus, some idea of the relative importance of the small-scale mixing and the bulk mixing rates may be obtained from a comparison of the t_m calculated from Equations (2) and (3) with t_3 estimated from Figure 5. For relatively nonviscous liquids such as water, the bulk mixing rate will probably be the limiting factor in the performance of stirred-tank reactors. As is evident from Equations (2) and (3), t_m is proportional to the viscosity to the 3/2 power; therefore, one expects the small-scale mixing rate to be the limiting factor in the performance of a stirred-tank reactor processing viscous liquids.

Equations (2) and (3) also show the advantage of the disk-and-vane turbine over the other turbine designs in applications where the small-scale mixing time is important. The disk-and-vane turbine with its relatively large power dissipation should produce a smaller scale of eddies and thereby a shorter small-scale mixing time than the other turbine designs for a given diameter turbine and a given rotational speed.

Another approach to the problem of mixing in stirred-tank reactors is the application of a model of the mixing process in a stirred tank. VandeVusse (*12*) proposed such a model which shows some of the characteristics that were observed in this study.

VandeVusse pictures the agitator in a stirred tank as a pump circulating the fluid in the tank around flow paths starting and ending at the impeller. The feed and overflow streams are assumed to be located on separate flow paths, and a flow path is assumed to be equivalent to a given

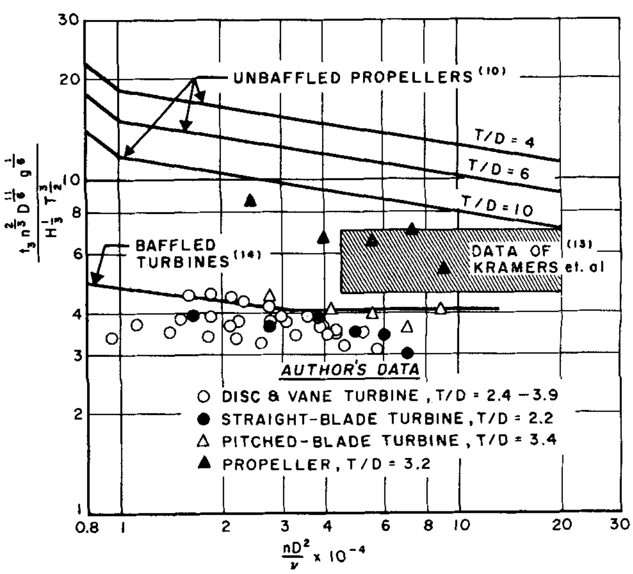

Fig. 5. Comparison with mixing time data of other investigators.

TABLE 3. MIXING TIMES DERIVED FROM VANDEVUSSE'S MODEL

N	t_1/τ	t_2/τ	t_3/τ	C_m/C_a
2	0.026	—	0.750	1.0
3	0.10	1.210	0.735	1.004
4	0.17	0.985	0.695	1.027
5	0.22	0.925	1.07	1.067
6	0.26	0.905	1.14	1.114
7	0.30	0.900	1.17	1.17
8	0.33	0.902	1.55	1.22
9	0.36	0.905	1.63	1.28
10	0.38	0.910	1.66	1.33
11	0.40	0.915	2.08	1.39
12	0.42	0.920	2.13	1.44
13	0.44	0.925	2.15	1.49
14	0.45	0.930	2.58	1.54
15	0.46	0.935	2.63	1.59

number of ideal mixing stages in series to characterize the mixing that occurs along the flow path.

VandeVusse simplified the model by assuming that all the flow paths could be characterized by an average flow path with N ideal mixing stages and a residence time, τ, along the path. VandeVusse derived equations for the response in the overflow stream to a pulse injection of tracer in the inlet stream for a continuous-flow stirred tank and a batch stirred tank. The shape of the response function depends upon the feed rate, the numbers of ideal stages along the average flow path, and the average residence time of the liquid along the average flow path. Of course, for batch mixing, the response function depends upon only the latter two parameters.

The shape of the calculated response curves for a batch-mixing tank resembles the observed response as illustrated in Figure 2. If the model is applied to the data reported in Figures 3 and 4, the number of mixing stages, N, may be found from the concentration ratio, C_m/C_a, and the residence time, τ, may be found from any of the mixing times t_1, t_2, or t_3 and the value of N.

The relationships between the parameters N and τ and the quantities t_1, t_2, t_3, and C_m/C_a were calculated from VandeVusse's equations for batch mixing and are summarized in Table 3. The mixing time t_1/τ was taken as the time for which the dimensionless concentration C/C_a equaled 0.10, and the terminal mixing time t_3/τ was taken as the largest time for which C/C_a is outside the range 0.95 to 1.05. These definitions of t_1/τ and t_3/τ were chosen to be consistent with the definitions for the corresponding experimental quantities.

The terminal mixing time t_3/τ appears to be a discontinuous function of N because of the oscillatory nature of the response function and the definition of t_3/τ.

VandeVusse's model was applied to the author's data for disk-and-vane turbines by estimating the values of the parameters N and τ from the experimental values of C_m/C_a and t_2 using Table 3. Values of t_1 and t_3 were predicted from Table 3 and compared with the experimental values of t_1 and t_3. The predicted values of t_1 averaged about 35% lower than the observed values, and about 80% of the predicted values of t_3 were within ± 25% of the observed values with a maximum deviation of about 50%.

Therefore, VandeVusse's model represents the observed behavior of the stirred tank fairly well, and the reader is referred to his paper (*12*) for a discussion of the application of the model to chemical reactors. The parameters of the model may be determined from values of t_3 and C_m/C_a providing that these quantities can be estimated for large-scale stirred tanks. Figure 5 provides a means of estimating terminal maxing times; however, data are lacking for the effect of tank diameter and liquid depth on C_m/C_a. The coefficients listed in Table 2 suggest the following relationship:

$$\frac{C_m}{C_a} \propto \left(\frac{nD^2}{\nu}\right)^{-1/6} \left(\frac{T}{D}\right)^{2/3} \quad (4)$$

Equation (4) is recommended as a first approximation of the effect of scale up on C_m/C_a for stirred tanks geometrically similar to those used in this study.

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NOTATION

b_o, b_1 = regression coefficients, Table 2
C_m, C_a = concentrations, defined in Figure 2, ML^{-3}
D = impeller diameter, L
D_v = coefficient of molecular diffusion, L^2T^{-1}
g = acceleration of gravity, LT^{-2}
H = liquid depth, L
l_s = scale of smallest eddies, L
N = number of mixing stages in stirred-tank model
n = revolutions per unit time of impeller, T^{-1}
T = tank diameter, T
t_1, t_2, t_3 = mixing times defined in Figure 2, t_3, also called *terminal mixing time*, T
t_m = microscopic mixing time defined by Equation (3), T
ϵ = rate of energy dissipation per unit mass, L^2T^{-3}
ν = kinematic viscosity, L^2T^{-1}
τ = average residence time in a flow path in stirred-tank model, T
$\propto$ = a proportional relationship

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